MATH 90 – CHAPTER 6 Name: .

6.1 GCF and Factoring by Groups

- Need To Know
  - Definitions
  - How to factor by GCF
  - How to factor by groups

The Greatest Common Factor

Factoring means to write a number as product.

_____ a polynomial means to _____a polynomial as a _____.

The **Greatest Common Factor** of a polynomial is the largest monomial that divides each term of the polynomial.

Examples – Identify the GCF then **FACTOR**:

$6x + 12$ $8y^4 + 12y^3 - 4y^2$

The Greatest Common Factor

Factor each polynomial

$ab + ac$

$7x + 7$

$10x^6 + 15x^4$

$-35a^6z^4 + 14a^4z^7 - 21a^3z^7$

You Try

$25x^4 + 35x^3$

$-20x^8 - 12x^7 + 4x^6$

GCF's may not be Monomials

Factor:

x(stuff) + 2(stuff)

$x(x - 6) + 2(x - 6)$

$a(z + 11) - w(z + 11)$

Factoring by Groups

Factor by Groups

1. No GCF and 4 terms
2. _____
   _____
3. _____
   _____

Factor by Groups:

1) $a^3 - 7a^2 + 4a - 28$

2) $8x^3 + 12x^2 - 14x - 21$

3) $20g^3 - 4g - 35g^2 + 7$

end

6.2 Factoring Trinomials

- Need To Know
  - Diamond puzzle
  - Idea of reverse FOIL
  - Factoring Trinomials

Diamond Puzzle

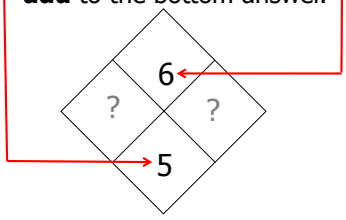
A puzzle that builds mental skills needed for factoring.

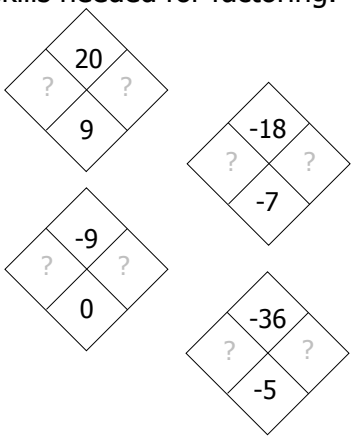
Find two numbers that:

multiply to the top answer

and

add to the bottom answer.





The Foil Method in Reverse

Multiply:

$(z - 6)(z + 2)$

Short Cut: **FOIL**

F – first terms mult.

O – outer terms mult.

I – inner terms mult.

L – last terms mult.

Factor:

$z^2 - 4z - 12$

Reverse FOIL

1. Write out parentheses ( )( )
2. _____pair that
   - multiplies to the last and
   - adds to the middle term.
3. _____ and check for a match on the add term.
4. Guess, Check "O" & "I", Revise

Factoring Trinomials

Factor:

$x^2 + 5x + 6$

Factor:

$z^2 - 7z + 6$

Factoring Trinomials w/ GCF

Factor:
$2a^2 - 2a - 24$

Factor:
$3z^3 + 18z^2 + 15z$

Factoring Trinomials w/ 2 Vars

Factor:
$x^2 + 7xy + 12y^2$

Factor:
$z^2 - 3za - 10a^2$

end

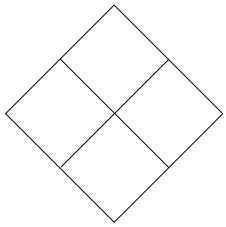
6.3 More Factoring Trinomials

- Need To Know
  - Review diamond puzzle
  - Methods of factoring Trinomials
    1. Guess, check and revise
    2. The Grouping Method
       (see the book for this technique)

Diamond Puzzle – Limitations

Factor:

$x^2 - 3x - 10$



Factor:

$2a^2 + 7a + 6$

Factoring Trinomials w/ $a \neq 1$

Factor:

$2a^2 + 7a + 6$

Reverse FOIL

0. Standard Form; Factor GCF
1. Write out parentheses ( )( )
2. Pick the sign pair that
   - multiplies to the last and
   - adds to the middle term.
3. _____
4. _____
5. Guess, _____Revise

Factoring Trinomials w/ $a \neq 1$

Factor:

$21y^2 - 70y - 56$

Reverse FOIL

0. Standard Form; Factor GCF
1. Write out parentheses ( )( )
2. Pick the sign pair that
   - multiplies to the last and
   - adds to the middle term.
3. List all factorings of first term
4. List all factorings of last term
5. Guess, Check "O" & "I", Revise

Factoring Trinomials w/ $a \neq 1$

Factor:

$15t^2 - 34t + 8$

Reverse FOIL

0. Standard Form; Factor GCF
1. Write out parentheses ( )( )
2. Pick the sign pair that
   - multiplies to the last and
   - adds to the middle term.
3. List all factorings of first term
4. List all factorings of last term
5. Guess, Check "O" & "I", Revise

Factoring Trinomials w/ $a \neq 1$

Factor:

$10p^2 + 5pq - 30q^2$

Factor:

$-14t^4 + 19t^3 + 3t^2$

Conclusion

Ways to Factor Polynomial

1. By Greatest Common Factor (GCF)
2. By Grouping
3. Factor Trinomials
   - Guess, check and revise
   - The Group Method (see book)

end

6.4 Special Factoring

Need To Know

- Recall some formulas
- Factoring the difference of two squares
- Factoring perfect square trinomials
- Formulas to Remember:

1. $(A + B)(A - B) =$ _____
2. $(A + B)^2 =$ _____
3. $(A - B)^2 =$ _____

Factoring a Difference of Squares

Formulas to Know

$A^2 - B^2 = (A + B)(A - B)$

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Factor:

$9x^2 - 25$

$49z^2 - 1$

Factoring a Difference of Squares

Formulas to Know

$A^2 - B^2 = (A + B)(A - B)$

$\mathbf{A^2 + B^2}$ _____

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Factor:

$x^2 + 25$

$a^4 - 16$

Factor- Perfect Square Trinomials

Formulas to Know

$A^2 - B^2 = (A + B)(A - B)$

$A^2 + B^2$ CAN'T Factor

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Factor:

$m^2 + 12m + 36$

$4x^2 + 12x + 9$

Factor- Perfect Square Trinomials

Formulas to Know

$A^2 - B^2 = (A + B)(A - B)$

$A^2 + B^2$ CAN'T Factor

$A^2 + 2AB + B^2 = (A + B)^2$

$A^2 - 2AB + B^2 = (A - B)^2$

Factor:

$16x^2 + 49 - 56x$

$18x^2 - 60x + 50$

Conclusion

Ways to Factor Polynomial

1. By Greatest Common Factor (GCF)
2. By Grouping
3. Factor Trinomials
   - Guess, check and revise
4. Factoring with Formulas
   - $A^2 - B^2 = (A + B)(A - B)$
   - $A^2 + B^2$ CAN'T Factor
   - $A^2 + 2AB + B^2 = (A + B)^2$
   - $A^2 - 2AB + B^2 = (A - B)^2$

6.5 Special Factoring - CUBES

Need To Know

- Factoring a Difference of Cubes
- Factoring a Sum of Cubes
- Using Formulas

Factoring Cubes

Factor $x^3 - 64$

Formulas for Factoring Cubes

Factoring a Sum or Difference of Two Cubes

_____

_____

When factoring a sum or difference of cubes,
it can be helpful to remember that
$2^3 = 8$, $3^3 = 27$, $4^3 = 64$, $5^3 = 125$, $6^3 = 216$, …$10^3 = 1000$

The list of number: _____…
are called perfect cubes.

Practice

Write an equivalent expression by factoring:

$x^3 - 27$ $11c^5 + 88c^2$

Practice

Write an equivalent expression by factoring:

$w^6 + 125z^3$ $y^3z^{12} - 1$

Factoring by Using Formulas

Sum of cubes:

$A^3 + B^3 =$ _____

Difference of cubes:

$A^3 - B^3 =$ _____

Difference of squares:

$A^2 - B^2 =$ _____

Sum of two squares – **NO FORMULA**

$A^2 + B^2$ **can't be factored**

6.6 Factoring Review

- Need To Know
  - Factoring Review
  - Practice

Ways to Factor Based on Terms

A. Check for GCF factoring (Always do first!)

B. Look at the Number of Terms

- Two Terms – Formulas

  $A^2 - B^2 = (A + B)(A - B)$ _____

  $A^2 + B^2$ can NOT factor _____

- Three Terms

  Guess, check, and revise

  Formulas: $A^2 + 2AB + B^2 = (A + B)^2$

  $A^2 - 2AB + B^2 = (A - B)^2$

- Four Terms

  By Grouping

C. Always Factor Completely – Try to factor more.

Factoring Practice

WAYS TO FACTOR

A. Factor GCF

B. Look at the Number of Terms

Two Terms – Use Formula

1. $A^2 - B^2 = (A + B)(A - B)$
2. $A^2 + B^2$ can't factor
3. $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
4. $A^3 + B^3 = (A + B)(A^2 - AB + B^2)$

Three Terms

Guess, check, and revise

Formulas

1. $A^2 + 2AB + B^2 = (A + B)^2$
2. $A^2 - 2AB + B^2 = (A - B)^2$

Four Terms

By Grouping Method

C. Always Factor Completely

Try to factor more!

Factor:

$3x^6 - 243x^2$

$y^5 + 8y^3$

$y^5 + 8y^2$

Factoring Practice

WAYS TO FACTOR

A. Factor GCF

B. Look at the Number of Terms

- Two Terms – Use Formula
  $A^2 - B^2 = (A + B)(A - B)$
  $A^2 + B^2$ can't factor
  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
- Three Terms
  1. Guess, check, and revise
  2. Formulas
     $A^2 + 2AB + B^2 = (A + B)^2$
     $A^2 - 2AB + B^2 = (A - B)^2$
- Four Terms
  By Grouping Method

C. Always Factor Completely
Try to factor more!

Factor:

$w^6 - 64$

$15a^2b^2 - ab - 2$

$2x^5 + 20x^4 + 50x^3$

Factoring Practice

WAYS TO FACTOR

A. Factor GCF

B. Look at the Number of Terms

- Two Terms – Use Formula
  $A^2 - B^2 = (A + B)(A - B)$
  $A^2 + B^2$ can't factor
  $A^3 - B^3 = (A - B)(A^2 + AB + B^2)$
- Three Terms
  1. Guess, check, and revise
  2. Formulas
     $A^2 + 2AB + B^2 = (A + B)^2$
     $A^2 - 2AB + B^2 = (A - B)^2$
- Four Terms
  By Grouping Method

C. Always Factor Completely
Try to factor more!

Factor:

$-4x^4 + 12x^3 + 40x^2$

$2a^5b + 6a^4b + 2a^3b$

$3ab + 9a + 2b + 6$

6.7 Solving Quadratic Equation

Need To Know

- Vocabulary and facts
- Solving quadratic equations

Vocabulary

Definition –
The **degree** of a polynomial in one variable is the exponent from the term with the highest power.

Fact –
The degree of an equation _____
_____.

The Principle of Zero Products

For real numbers a and b,

_____

Solve Polynomial Equations

Solve for x:
$(x + 3)(x + 10) = 0$

Solve for z:
$6(5z - 3)(z + 8) = 0$

Vocabulary

Definition – a **quadratic equation** is one that can be put in the form $ax^2 + bx + c = 0$ where a, b, & c are real numb. ($a \neq 0$).

$ax^2 + bx + c = 0$ is a quadratic in _____.

Constant Term

Linear Term

Quadratic Term

Solve Quadratic Equations

Steps to Solve Q.E.

1. _____
2. _____
   _____
3. _____
   _____
4. Check solutions

Solve for x:

$x^2 + 6x + 8 = 0$

Solve Quadratic Equations

Steps to Solve Q.E.

1. Set up must = 0
2. Factor completely
3. Solve each factor for zero
4. Check solutions

Solve for x:

$2x^2 + 5x = 3$

Solve Quadratic Equations

Solve for a:

$49a^2 - 16 = 0$

Solve for z:

$30z^2 = -12z$

Solve Quadratic Equations

Solve for w:

$w^2(2w - 1) = 3w$

Solve for x:

$(2x-5)(3x^2+29x+56)=0$

end

6.8 Applications

Need To Know

- Recall guide lines to solve word problems
- Recall tool to solve word problems
- Solve word problems with 2 unknowns

Guide Lines for Word Problems

Blueprint for Solving

1. Read and understand the problem (# of unknowns)
2. Assign variables and write down the meaning of the variable
3. Write an equation
4. Solve the equation
5. Write down your answer using a complete sentence
6. Reread and check your solution

Tools to Reveal the Equation

1. Use keywords
2. Draw a picture
3. Make up a simpler problem
4. Make tables of numbers and look for patterns
5. Use charts to organize your information
6. Make a guess
7. Use a verbal model

Apply

A number is two less than its square. Find all such numbers.

Steps

1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools

1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Apply

The product of two consecutive odd integers is 63. Find the integers.

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Apply

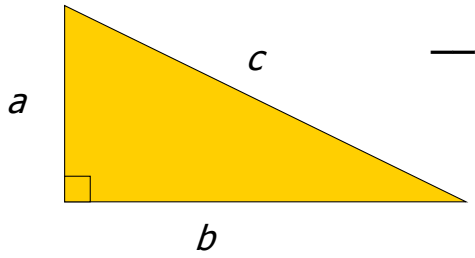
The length of a rectangle is 2 more than twice the width. The area is 60 in^2. Find the dimensions.

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model

Pythagoras and The Theorem

Pythagorean Theorem:
In any right triangle,
if *a* and *b* are the lengths of the legs and
c is the length of the hypotenuse,



_____.

Apply

Steps
1. Familiarize
2. Translate
3. Carry out
4. Check
5. State answer

Deb's garden was a right triangle.

One leg of the triangle was formed by a 400-ft wall. The hypotenuse of the triangle was 200 ft longer than the other leg. What are the dimensions?

Tools
1. Keywords
2. Drawing
3. Simpler problem
4. Tables/Patterns
5. Charts
6. Guess
7. Verbal Model